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Fiscal Deficits as a Source of Boom and Bust under a Common Currency*

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Abstract

We investigate in depth, using predominantly analytical rather than numerical methods, the mechanisms triggered by a one-off debt-financed fiscal deficit in a small open economy with a shared currency. The economy incorporates staggered price setting and overlapping generations. Unsurprisingly, these cause the impact effect to be a boom, in the sense of price inflation and a positive output gap. However, contrary to what normally happens in New Keynesian models without extraneous dynamics, the boom later inevitably turns into a bust, i.e. price deflation and a negative output gap. Therefore, in this setting, while short-run Keynesian deficit-based fiscal stimulus 'works', it also provokes a medium-run 'backlash' in aggregate activity.

JEL Classification

E62, F45, H62

Keywords

staggered prices, overlapping generations, small open economy, currency union, fiscal deficits, government debt, output gap

1. Introduction

The recent resurgence of interest in fiscal policy in New Keynesian settings has elicited agreement that a crucial factor in its effectiveness is the way it interacts with monetary policy.[1] In a closed economy, or an open economy with an independent monetary policy, any fiscal policy change affects aggregate variables some of which are of concern to the monetary authority, and how monetary policy reacts (or not) to such variables is critical for the effect of fiscal policy. However, this sensitivity is avoided in a small open economy under a common currency because, there, a country's fiscal policy has a negligible effect on aggregate variables at the level of the currency union, so that the question of how the monetary authority reacts does not arise.[2] The independence of fiscal policy's effects from the monetary policy regime thus gives this case a special character from a theoretical perspective. It is also a very relevant case to study from a practical perspective. In a currency union, a country has given up its monetary policy as an independent instrument, so that its fiscal policy becomes potentially the most important item in its macroeconomic policy toolkit. It is hence vital to have a deep understanding of the effects of its fiscal policy on the state of its business cycle. Moreover, if we consider a regional fiscal authority within a country, the situation it faces is essentially the same, so that our paper also be seen as an analysis of how a regional fiscal deficit affects the regional level of economic activity.

Considerable recent attention has already been given to the case of the government spending multiplier, and moreover nearly always the 'balanced-budget' multiplier. In the present contribution we wish to focus instead on the role of fiscal deficits. To separate this from the effects of changes in government spending, we focus on deficits brought about by tax cuts.[3] In the history of thinking about Keynesian aggregate demand management through fiscal means, deficits and the government debt associated with them have been at least as important as government spending. However, to give a role to debt and deficits per se in a dynamic

[1] See, for example, Woodford (2011) and Ascari and Rankin (2013).
[2] If the issuance of debt increases the risk premium on such debt then it may affect, especially, the long-term interest rate; but in our formal model here we abstract from default risk.
[3] Deficits brought about by spending increases can be analysed as the sum of a tax-financed spending increase and a debt-financed tax cut. Since these two policies operate through fairly different mechanisms, separating them provides the clearest understanding of the effects at work.

general equilibrium context we need a theoretical framework which avoids Ricardian Equivalence. Whether Ricardian Equivalence should indeed be avoided can of course be debated. However, recent time series econometric evidence generally shows large, negative, tax multipliers on GDP.[4] Although such work is silent on whether this is due to incentive effects of marginal tax rate changes, or to intertemporal shifting of households' disposable income which is not offset for other reasons by changes in their saving behaviour, the latter remains a strong candidate as an explanation. It is the latter which is generally meant by 'non-Ricardian' behaviour by households, and this is the type of behaviour whose implications we investigate further here.

There are several possible approaches to modelling non-Ricardian behaviour. A commonly used one in recent work is to assume a fraction of 'hand-to-mouth' households. Examining more than one approach at once is not easily feasible owing to the modelling investment required. The approach we adopt here is one of overlapping generations (OLGs). This has the merit of being grounded in the simple and undeniable fact that human beings have finite lives. Specifically, we utilise Blanchard's (1985) 'uncertain lifetimes' version. The OLG approach generates an allocative role for financial assets, unlike the hand-to-mouth approach, in that trade in such assets permits heterogeneity in consumption levels across households. We believe the implications of such a role for the effects of government debt in a business-cycle context have been under-explored. Nevertheless, the OLG framework has been employed in New Keynesian open-economy models by a few authors, including Ganelli (2005) and Leith and Wren-Lewis (2006, 2008).[5] Relative to Ganelli the innovations of the current paper are that the dynamics of price-setting are more thoroughly treated and that we consider a common currency not a floating exchange rate.[6] Relative to Leith and Wren-Lewis, the difference is that we focus on dissecting the mechanics of fiscal multipliers rather than on stability and international coordination questions.

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[4] See Ramey (2019).

[5] It has also been used in a closed-economy New Keynesian framework by, amongst others, Devereux (2011), Annichiarico et al. (2012) and Ascari and Rankin (2013).

[6] Ganelli (2005) assumed one-period nominal rigidities, as in the the 'New Open-Economy Macroeconomics' approach of Obstfeld and Rogoff (1995, 1996), whereas here we assume staggered price setting in the style of Calvo (1983).

Our main finding is that, while in this setting a fiscal deficit does, as might be expected, cause a short-run boom in the form of a positive 'output gap', such a boom has unorthodox features. Most notably, the boom is always followed by a 'bust', or negative output gap. In other words, the business-cycle response of the economy is non-monotonic: after impact, instead of the boom simply decaying to zero, it goes into reverse. The output gap disturbance does eventually fade to zero, but from below, not above. Therefore, although a fiscal deficit has a short-run benefit because it generates a cyclical upturn, it also has a medium-run cost through inducing a subsequent downturn. This is not a standard and well-documented feature of Keynesian stimulus. More typically, the expansionary impact fades away monotonically as prices have time to adjust. It is true that, in models with multiple sources of dynamics, like empirical DSGE models, a delayed perverse reaction to a demand stimulus can sometimes occur; but in our analysis it occurs in a setting which has the minimum necessary sources of dynamics to capture an expansionary impact effect in the first place. Such a business-cycle backlash is something which policymakers obviously need to be wary of. It does not necessarily mean that they should avoid deficit-based fiscal stimulus, but it does imply that they need to design stabilisation policy to mitigate its unwelcome downstream consequences. Second, we find that inflation also exhibits a boom-bust cycle, with the deflation phase starting while the output gap boom is still in progress. On average the fiscal deficit shock causes deflation, not inflation. This runs contrary to the common idea that an aggregate demand stimulus must raise prices.[7] We demonstrate that the boom-bust cycle and the deflation are robust phenomena, occurring for all parameter values. We go on to explain why they are by-products of the same features which make the fiscal stimulus policy work at all.

The mechanism which causes boom to be followed by bust in our model turns out to be the temporary nature of the boost to aggregate demand which an increase in government debt provides, even though the debt increase is itself permanent. The boost to aggregate demand is temporary because, in a small open economy with overlapping generations, an injection of government debt gradually crowds out net foreign assets. In the long run, the loss of net foreign

[7] A recent empirical study which also finds that a fiscal stimulus lowers rather than raises prices is by Jorgensen and Ravn (2018).

assets more than offsets the increase in government debt in terms of its effect on aggregate demand. In the face of prices which are sluggish to adjust, the later decline in aggregate demand causes a recession, even though this decline is fully anticipated. We highlight this mechanism by contrasting it with the case of a balanced-budget increase in government spending. This causes a boom which fades away monotonically, without turning into a bust. The reason is that the boost to aggregate demand in this case is permanent, since it does not cause crowding out of net foreign assets.

Our paper is a theoretical exploration rather than an attempt to match real-world data. We are interested in acquiring a deep qualitative understanding of the structural mechanisms through which fiscal deficits may affect macroeconomic variables. For this reason we use the simplest possible specification which still incorporates our desired ingredients of staggered price setting and overlapping generations. This enables us to derive our main conclusions analytically, including results about the detailed dynamics. We also focus on a very basic type of fiscal experiment, namely a one-period debt-financed tax cut, thereby avoiding clouding the picture with additional sources of dynamics of a kind which could result from more 'realistic' fiscal feedback rules. Nevertheless, we do provide a brief numerical illustration of possible time paths, for parameter values which we consider to be relevant empirically. This confirms that the 'bust' can indeed be of significant magnitude. As regards evidence of such boom-bust behaviour in practice, direct testing of this is not something that we attempt. However SVAR estimates of fiscal multipliers exhibit a wide variety of patterns of dynamic response to fiscal shocks. Some of these do indeed display a boom-bust pattern: see, for example, Italy's response to a tax revenue shock as estimated by Afonso and Sousa (2012).[8]

The amount of literature so far devoted to studying the role of government deficits and debt within an open-economy New Keynesian dynamic general equilibrium framework is not large. The approach of assuming that a fraction of households are 'hand-to-mouth' agents who have no access to asset markets has been used by, for example, Corsetti et al. (2013), Erceg and Lindé (2013) and Farhi and Werning (2016).[9] However such work has not drawn attention

[8] This is illustrated in their Figure 9(b).
[9] Examples for a closed economy are Galí et al. (2007) and McManus (2015).

to the possibility of fiscal deficits causing a delayed 'bust', as we do here. The approach of assuming that only distortionary taxation is available has been taken by Ferrero (2009). While not denying the potential importance of distortionary taxation, in the present paper we keep the focus on how even lump-sum taxation, in combination with government debt, can have a macroeconomic impact – which is to say through its intergenerational redistribution role.

The paper proceeds, in Section 2, by laying out the microeconomic assumptions of the model. In Section 3 we draw out the implied macroeconomic structure. Section 4 then uses this to analyse the effects of a one-off fiscal deficit, and Section 5 concludes.

2. The Microeconomic Elements

We consider a 'small' country which takes world output and interest rates as given, and where international trade in goods and assets is frictionless. The country also takes world prices of foreign-produced goods as given. However domestically-produced goods are differentiated from foreign-produced goods, so that prices of the former may be affected by events at home. The potentially 'Keynesian' nature of the macroeconomic equilibrium arises from assuming staggered price setting in the style of Calvo (1983). This is embedded in a dynamic general equilibrium framework by combining it with monopolistically competitive firms, in the manner of Woodford (2003), Galí (2015), and many other authors. On the other hand, the absence of Ricardian Equivalence, which generates the scope for government debt and deficits to have real effects, arises from assuming overlapping generations in the style of Blanchard (1985).

2.1 Household behaviour

Domestic households supply labour (L) and consume a foreign good (C^F) and a composite domestic good (C^H). We assume the economy is 'cashless' in that households' holdings of real balances are sufficiently small that, as a reasonable approximation, their demand for money can be neglected. The key assumption of Blanchard (here adapted for discrete time) is that each household has an exogenous probability of death, 1-q, per period of time (where $0 < q \leq 1$). The size of the population is normalised to 1, so that in every period 1-q existing households die and 1-q new households are born. By varying q we can vary the expected lifetime (= 1/[1-q]) of a household. An insurance market is assumed in which the household agrees to cede all its financial wealth to the insurance company in the event of its death, in return for which it receives an 'annuity' at a gross rate $1/q$ on its financial wealth in every period in which it remains alive. A household is born with zero financial wealth, but over its life it will generally accumulate financial wealth (or, alternatively, debt). Households of different ages will therefore have different wealth and consumption levels. We use 's' to denote the birth-period of a household. Households may also hold (or issue) bonds, (F^N, in nominal terms) which pay a nominal interest rate i. The bonds held by domestic households are issued either by the government or by foreigners. Since there is no aggregate uncertainty, such bonds are perfect substitutes from households' viewpoint and hence they all pay the same interest rate.

Given the foregoing, the dynamic optimisation problem of a household may be written as:

maximise $$\Sigma_{t=n}^{\infty}(\beta q)^{t-n}\left[\gamma \ln C_{s,t}^{H}+(1-\gamma)\ln C_{s,t}^{F}+\psi \ln(1-L_{s,t})\right] \tag{1}$$

subject to $$P_t^H C_{s,t}^H + P_t^F C_{s,t}^F + F_{s,t+1}^N \;=\; (1/q)(1+i_t)F_{s,t}^N + W_t L_{s,t} + \Pi_t - T_t, \quad \text{for } t = n,\ldots,\infty. \tag{2}$$

Here, $0 < \beta, \gamma < 1$; $\psi > 0$; $s \le n$. Note that all households face the same lump-sum tax, T_t, and receive the same share of profits, Π_t, from the monopolistic firms. (1) gives the expected discounted lifetime utility of a household, accounting for the fact that the probability of survival from one period to the next is q. To enable aggregation across households of different ages we need preferences to be homothetic. This is achieved here by adopting a simple logarithmic utility function.[10]

Clearly, we can separate the static problem of optimally allocating consumption spending between home and foreign goods from the dynamic optimisation problem. To do this, define the sub-utility over goods consumption as a whole as:

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[10] One consequence of (1) is that there will generally be some, sufficiently old, households, whose labour supply, $L_{s,t}$, is negative. This phenomenon and how it might be avoided are discussed in Ascari and Rankin (2007). It is unappealing on grounds of microeconomic realism. However, since the negativity of some labour supplies is not a factor on which the macroeconomic properties of the model hinge, we choose to accept it here.

$$(C_{s,t}^H)^\gamma (C_{s,t}^F)^{1-\gamma} \equiv C_{s,t}. \tag{3}$$

Maximising this subject to a given total nominal spending on goods, $P_t^H C_{s,t}^H + P_t^F C_{s,t}^F = I_{s,t}$, leads to the following demand functions for home and foreign goods:

$$C_{s,t}^H = \gamma I_{s,t} / P_t^H, \qquad C_{s,t}^F = (1-\gamma) I_{s,t} / P_t^F. \tag{4}$$

The maximised value of $C_{s,t}$ is then $I_{s,t} / P_t$, where P_t is the domestic consumer price index,

$$P_t \equiv \gamma^{-\gamma}(1-\gamma)^{-(1-\gamma)}(P_t^H)^\gamma (P_t^F)^{1-\gamma}. \tag{5}$$

$C_{s,t}$ is thus also the real value of spending on all goods by the household, or its 'total composite consumption'. Note that the intertemporal relative price of $C_{s,t}$ is the real interest rate:

$$1 + r_t \equiv (1 + i_t) P_{t-1} / P_t. \tag{6}$$

We may now return to the full dynamic optimisation problem and re-express it in terms of $C_{s,t}$. Solving it then yields two first-order conditions which will constitute key equations of the model. Since these are linear in $(C_{s,t}, L_{s,t})$ and their coefficients are independent of s, they can also be expressed in terms of the aggregate counterparts of these generation-specific variables. We then have:

$$L_t = 1 - \psi \frac{P_t}{W_t} C_t, \tag{7}$$

$$C_{t+1} = (1 + r_{t+1})\beta C_t - \frac{1 - q\beta}{1 + \psi}(1/q - 1)(1 + r_{t+1}) F_{t+1}. \tag{8}$$

Note that, for any variable $X_{s,t}$, the corresponding aggregate is $X_t \equiv \Sigma_{s=-\infty}^{t} (1-q) q^{t-s} X_{s,t}$. (7) is the labour supply function, showing that desired labour supply is positively related to the real wage and negatively to consumption. (8) is the 'consumption Euler equation'. As in Blanchard (1985), this differs from its generation-specific counterpart by the presence of F_{t+1}, where F_{t+1} is the real value of households' bond (or 'financial') wealth, F_{t+1}^N / P_t.[11] Observe

[11] Unlike (7), (8) is not obtained simply by replacing generation-specific variables by aggregate variables in the corresponding first-order condition of an individual household. Its derivation also makes use of the expression for an individual household's consumption as a function of its total lifetime wealth.

that if q = 1 (zero probability of death), F_{t+1} drops out of (8). (8) then reduces to the Euler equation familiar from models with infinitely-lived agents. More generally, however, q < 1, in which case F_{t+1} has a negative effect (if F_{t+1} > 0) on the growth rate of aggregate consumption. This 'generational turnover effect' arises from the fact that, every period, some already-living agents are replaced by newborn agents. The newborn, who have no financial assets, have lower consumption than the already-living, since the latter have had time to accumulate such assets. This therefore tends to reduce aggregate consumption growth. Alternatively viewed, (8) says that, for a given expected value of C_{t+1} and r_{t+1}, an increase in F_{t+1} will increase C_t. Such dependence of current aggregate consumption, not only on expected future aggregate consumption and the real interest rate, but also on the country's aggregate stock of financial assets, is the key feature contributed by overlapping generations.

Above we referred to C^H as consumption of a 'composite' domestic good. Specifically, we assume that the home economy produces a continuum of varieties of good, each indexed by z, where $z \in [0,1]$. $C_{s,t}^H$ is a CES sub-utility function over these:

$$C_{s,t}^H = [\int_0^1 (C_{s,t}(z))^{(\theta-1)/\theta} dz]^{\theta/(\theta-1)}, \tag{9}$$

where θ (> 1) is the constant elasticity of substitution. The household faces the sub-problem of maximising (9) subject to a given nominal expenditure on home goods, $I_{s,t}^H$, where $I_{s,t}^H = \int_0^1 P_t(z) C_{s,t}(z) dz$. This leads to a constant-elasticity demand function for any variety, z:

$$C_{s,t}(z) = [P_t(z) / P_t^H]^{-\theta} I_{s,t}^H / P_t^H , \tag{10}$$

where P_t^H is the price index $[\int_0^1 (P_t(z))^{1-\theta} dz]^{1/(1-\theta)}$. The maximised value of $C_{s,t}^H$ is then $I_{s,t}^H / P_t^H$.

2.2 Firm behaviour

Domestic goods are produced by an industry which is monopolistically competitive. Each variety, z, is the output of a single firm with a production function $Y_t(z) = [L_t(z)]^{\sigma}$, where 0 < $\sigma \leq 1$ and $L_t(z)$ is the labour input. Labour is homogeneous and traded in a perfectly competitive, flexible-wage, market.

The total private domestic demand for any variety, z, may be found by summing the demand function (10) across all domestic households. This leads to a function of a similar form to (10) but in which the 'shift parameter', $I_{s,t}^H / P_t^H$, is replaced by total domestic consumption of the 'composite' home good, C_t^H. We furthermore assume that there is an analogous foreign demand function for the domestic good variety, z, also having the price elasticity θ. The shift parameter for this foreign demand function will similarly be equal to total foreign consumption of the composite home good, which we denote as C_t^{H*}. Moreover, in the absence of any barriers to international arbitrage in goods, the price of good z in the foreign market must also be $P_t(z)$. Lastly, when the domestic government purchases home-produced goods, we shall assume that it allocates its spending across the varieties, z, in the same way as private agents, treating its total spending on the composite good, G_t^H, as exogenous. The global demand for good z is hence:

$$Y_t(z) \;=\; [P_t(z) / P_t^H]^{-\theta}[C_t^H + C_t^{H*} + G_t^H]. \tag{11}$$

Since firm z is infinitesimal relative to the whole economy, it takes the macroeconomic variables $(P_t^H, C_t^H, C_t^{H*}, G_t^H)$ as given when choosing its price and output.

Staggered price setting is introduced by using Calvo's (1983) assumption that firms may only adjust prices in periods when they receive a random signal permitting this, and that in other periods they must keep prices fixed. 1-α is the exogenous probability of receiving such a signal. The price chosen by any firm which is permitted to adjust its price in period t is the 'new' price, denoted as X_t. Since the optimisation problem of a firm under these assumptions is very familiar from the expositions of Woodford (2003), Galí (2015), and many others, we will simply present the solution which results for the new price, namely:

$$X_t \;=\; \frac{\theta}{\theta - 1}\,\frac{\Sigma_{j=0}^{\infty}\alpha^j \Delta_{t,t+j}[\bar{Y}_{t+j}(z)/P_{t+j}]\dfrac{1}{\sigma}W_{t+j}[\bar{Y}_{t+j}(z)]^{1/\sigma - 1}}{\Sigma_{j=0}^{\infty}\alpha^j \Delta_{t,t+j}[\bar{Y}_{t+j}(z)/P_{t+j}]}. \tag{12}$$

Here $\Delta_{t,t+j} \equiv (1+r_{t+1})^{-1}(1+r_{t+2})^{-1}...(1+r_{t+j})^{-1}$ is the intertemporal discount factor used by the firm,[12] and $\bar{Y}_{t+j}(z) = (X_t / P^H_{t+j})^{-\theta}(C^H_{t+j} + C^{H*}_{t+j} + G^H_{t+j})$ is the demand for its product in period $t+j$ contingent on the firm having last received a price-adjustment opportunity in period t. As is standard, this equation says that the new price is given by a mark-up ($\theta/[\theta-1]$) over a weighted average of current and expected future marginal costs.

The formula for the index of prices of all domestically-produced goods, P^H_t, was given above, in conjunction with equation (10). Combining this with the assumption of Calvo-style price staggering, we obtain a well-known relationship between the price index and lagged values of the 'new' price:

$$P^H_t \;=\; [(1-\alpha)\Sigma^{\infty}_{j=0}\alpha^j X^{1-\theta}_{t-j}]^{1/(1-\theta)}. \tag{13}$$

Together, (12) and (13) form the key equations of the price-setting sector of the model.

2.3 Government behaviour

The government's budget constraint, expressed using the composite consumption good of domestic households as the numeraire, is:

$$P^H_t G^H_t / P_t \;-\; \tau_t \;+\; (1+r_t)D_t \;=\; D_{t+1}, \tag{14}$$

where D_t is the real value of government debt at the end of period t-1. We treat debt as being 'real' (or 'indexed') debt. Hence we abstract from effects caused by unanticipated price-level changes on the real value of nominally-denominated debt. G^H_t is real government spending on the composite home-produced good. Although, most of the time, we shall assume this is zero, we briefly consider a government spending increase in Section 4. τ_t ($\equiv T_t/P_t$) is the lump-sum tax levied on households.[13]

[12] Since there is no aggregate uncertainty in the model, the appropriate discount factor is simply the inverse of the product of the gross real riskless interest rates between t and $t+j$.
[13] An omission from the budget constraint is any seigniorage revenue. In a common currency area, such revenue in principle accrues to the area's central bank, which determines the area's monetary policy. A share of it could be distributed to the home country's government. However, under our assumption of a 'cashless' economy, even if the central bank did engage in monetary expansion (which we shall not consider here), the seigniorage revenue generated would be zero, since holdings of real balances by households are treated as negligible.

2.4 Market equilibrium conditions and some elementary macroeconomic relationships

For each domestically-produced good, *z*, the output must equal the sum of the total amounts purchased both at home and abroad. Hence $Y_t(z) = C_t(z) + C_t^*(z) + G_t(z)$, where $C_t^*(z)$ denotes the foreign consumption, and $G_t(z)$ denotes the purchases by the home government. An analogous relationship prevails at the level of the 'composite' home good (defined in (9)):

$$Y_t \;=\; C_t^H + C_t^{H*} + G_t^H \,. \tag{15}$$

Note that C_t^{H*} is a measure of the home country's exports.

It is useful to relate the private demands for the composite home-produced good to the terms of trade. We define the terms of trade as $\rho_t \equiv P_t^F / P_t^H$. Using the expression (5) for the domestic overall consumer price index, we then have:

$$P_t^H / P_t \;=\; \tilde{\gamma}\rho_t^{\gamma-1} \qquad \text{where } \tilde{\gamma} \equiv \gamma^{\gamma}(1-\gamma)^{1-\gamma} \,. \tag{16}$$

We can also readily show that:

$$C_t^H \;=\; \gamma C_t / (\tilde{\gamma}\rho_t^{\gamma-1}) \,. \tag{17}$$

For C_t^{H*}, the demand will depend on foreign households' preferences. We suppose these are also logarithmic over foreign goods and the composite home good (cf. (1)). This implies that foreign households will devote a constant fraction of their total nominal consumption spending to domestic goods. Analogously to (4) above, we may thus write the demand for C_t^{H*} as:

$$C_t^{H*} \;=\; I_t^{H*} / P_t^H \,, \tag{18}$$

where I_t^{H*} is the nominal budget which foreign households allocate to spending on home goods. Given that the home country is 'small' in the world economy, I_t^{H*} can be treated as exogenous. We can also write (18) as:

$$C_t^{H*} \;=\; K\rho_t \qquad \text{where } K \;=\; I_t^{H*} / P_t^F \,. \tag{19}$$

K is exogenous to the home country since, being small, it also takes P_t^F as given. We shall moreover treat it as time-invariant. Overall, then, aggregate demand for home-produced goods is related to the terms of trade, total domestic consumption and government demand by:

$$Y_t \;=\; \gamma C_t \,/\, (\tilde{\gamma}\rho_t^{\gamma-1}) \;+\; K\rho_t \;+\; G_t^H \,, \tag{20}$$

(obtained by substituting (17) and (19) into (15)). It is unambiguously increasing in ρ_t, capturing an expenditure-switching effect; and unambiguously increasing in C_t.

In the labour market, the money wage, W_t, adjusts flexibly to equate the aggregate supply of labour, given by (7), to firms' aggregate demand for it. Note that price-staggering generally causes a dispersion of prices across firms, and thereby a dispersion in their labour demands. Consequently, when labour demands are summed across firms, aggregate labour demand becomes:

$$L_t \;=\; s_t Y_t^{1/\sigma} \qquad \text{where } s_t \equiv \Sigma_{j=0}^{\infty}(1-\alpha)\alpha^j (X_{t-j} \,/\, P_t^H)_{t-j}^{-\theta/\sigma} \,. \tag{21}$$

s_t is a measure of price dispersion.[14] If there is no dispersion, as happens in a zero-inflation steady state, then $s_t = 1$. (21) is then just the inverse of an individual firm's production function, but here applied to aggregates. More generally, when there is dispersion, $s_t > 1$. However, when the model is log-linearised around a zero-inflation steady state, s_t drops out, so in fact it will play no role in our analysis below.

In the bond market, the equilibrium condition is that the aggregate demand for bonds by domestic households, F_t, should equal the supply of bonds by the home government, D_t, plus the supply of bonds by foreign residents. We denote the latter as V_t. They constitute the 'net foreign assets' of the home country as a whole.[15] Hence:

$$F_t = D_t + V_t. \tag{22}$$

[14] It was originally defined in this way, for staggered-price models, by Schmitt-Grohé and Uribe (2007).

[15] We here speak as if F_t, D_t and V_t are all positive, but they may also be negative, as occurs for V_t in the policy experiment studied below.

As earlier, all bond stocks are measured in real terms using the domestic household's composite consumption good as the numeraire.[16]

The accumulation or decumulation of net foreign assets arises from surpluses or deficits in the country's balance of payments. The trade surplus, expressed in units of domestic households' composite consumption, is:

$$B_t \;\equiv\; (P_t^H / P_t)C_t^{H*} \;-\; (P_t^F / P_t)C_t^F \,. \tag{23}$$

Replacing (C_t^{H*}, C_t^F) by their respective demand functions, and replacing the relative prices by writing them in terms of ρ_t, we have:

$$B_t \;=\; K\tilde{\gamma}\rho_t^{\gamma} \;-\; (1-\gamma)C_t \,. \tag{24}$$

(24) is the home country's 'net export demand function'. It shows that a worsening of the terms of trade (increase in ρ_t) boosts net exports by raising export demand, while an increase in domestic total consumption shrinks net exports by raising import demand. Given the trade balance, net foreign assets then evolve according to the standard identity:

$$V_{t+1} \;=\; (1+r_t)V_t \;+\; B_t \,. \tag{25}$$

$B_t + r_tV_t$ is the current account surplus on the balance of payments, so this says that net foreign assets increase or decrease over time according as there is a current account surplus or deficit (respectively).

Under a single currency, the nominal interest rate is the same at home and abroad. However, the same does not necessarily apply to the *real* interest rate, which is the intertemporal relative price of goods. In the case of the foreign-produced good, its intertemporal relative price is exogenous to the home country, since the latter is 'small' in world markets. Hence, defining the foreign real interest rate as $1+r_t^F \equiv (1+i_t)P_{t-1}^F / P_t^F$, r_t^F must be treated as given. The intertemporal relative price of home-produced goods, on the other hand, will generally be affected by events at home, since they are differentiated from foreign goods and so their prices are not completely tied down by world prices. Now, we previously

[16] As with government debt, we treat the debt of foreigners held by domestic residents as 'real', or 'indexed', debt.

defined the domestic real interest rate as the intertemporal relative price of domestic households' composite consumption (see (6)). Since home-produced goods make up a significant part of this, it follows that the domestic real interest rate is also not exogenous. Its relationship to the foreign real interest rate is given by:

$$1 + r_{t+1} \;=\; (1 + r_{t+1}^{F})(\rho_{t+1} / \rho_t)^{\gamma}\,, \tag{26}$$

as can be derived from the foregoing definitions. This is an 'uncovered interest parity' condition, but in real terms. It says that the domestic real interest rate can deviate from the foreign one to the extent that the country's terms of trade are expected to change between this period and the next.

3. The Macroeconomic Structure

In this section we assemble the above elements into an apparatus suitable for studying the full macroeconomic effects of changing government debt. We begin with an analysis of the economy's steady state equilibrium. We then present the log-linear approximation which will be used for the study of the dynamics. In doing so, we highlight a key property of the model, which is the separability of the sector consisting of net foreign assets and the trade balance. This is what makes it feasible to characterise the dynamics without relying on numerical simulations.

3.1 The zero-inflation steady state equilibrium

We assume the common currency area's monetary policy ensures zero foreign inflation. In the domestic economy the steady state then has to be one in which domestic inflation is also zero, or else the terms of trade would be permanently changing.

Note that in a zero-inflation steady state the staggered price setting has no real effects, because in the long run all prices have had time to adjust. Steady-state output is therefore essentially determined by 'supply-side' forces. In Appendix A, we show that the terms of trade and output then each have unique negative relationships to total domestic consumption (for given government spending, G^H):

$$\rho = \rho(C, G^H), \qquad Y = Y(C, G^H). \tag{27}$$

These arise because greater lifetime wealth of domestic households (for which C can be considered a proxy) simultaneously increases the demand for home-produced goods and reduces the supply of them. The reduced supply occurs because wealthier households demand more leisure and so withdraw labour supply. The combination of reduced supply and increased demand then pushes up the relative price of home goods in the world market, i.e. reduces ρ.

In a steady state it must also be the case that the country's terms of trade, ρ_t, are constant over time. The 'real UIP' condition, (26), then implies that the domestic real interest rate, r_t, has to equal the foreign real interest rate, r_t^F. The latter is exogenous, and henceforth we will also take it to be time-invariant, denoting it as r^F.

It must furthermore be true that the country's net foreign assets are unchanging in a steady state. This requires the current account of the balance of payments to be zero, or $B + r^F V = 0$. In other words (and assuming $r^F > 0$), if the country is a permanent net debtor ($V < 0$), it must run a permanent trade surplus ($B > 0$) in order to finance the interest on its foreign debt; and conversely if it is a net creditor. Given the relationship of the trade surplus to total consumption via the net export demand function (24), and also the relationship of the terms of trade to total consumption in (27), it is then not surprising to find that in a steady state a country with higher net foreign assets will have higher total consumption, as can be seen from:

$$-r^F V \;=\; K\tilde{\gamma}[\rho(C, G^H)]^{\gamma} \;-\; (1-\gamma)C\,. \tag{28}$$

A final steady-state relationship derives from the consumption Euler equation, (8). When total consumption is constant over time, this implies (along with $r_{t+1} = r^F$ and $F_t = D_t + V_t$):

$$V + D \;=\; \frac{1+\psi}{(1-q\beta)(1/q-1)}\left[\beta \;-\; \frac{1}{1+r^F}\right]C\,. \tag{29}$$

We can think of (29) as domestic households' steady-state demand for financial assets. It arises from the life-cycle pattern of asset accumulation which occurs under overlapping generations. To see this, note that if r (= r^F) > $1/\beta - 1$, then each household will choose an ever-rising path for its own consumption, since this is implied by its individual Euler equation. Given that, in a steady state, it faces a 'flat' path for its after-tax labour income, then to achieve the desired consumption trajectory the household has to continually accumulate financial assets during its lifetime. In a steady state with $r^F > 1/\beta - 1$, a typical household's financial assets thus grow steadily, starting from zero; but upon death they drop abruptly back to zero. In such a steady state the aggregate demand for financial assets is hence positive, and it is greater, the greater is r^F. This is what (29) shows. Conversely, if $r^F < 1/\beta - 1$, households accumulate debt during their lifetimes, and aggregate demand for financial assets is negative. Since both r^F and β are exogenous parameters, the sign of $\beta - 1/(1+r^F)$ is a matter of assumption. For present purposes it is convenient to focus on the case where $\beta = 1/(1+r^F)$. Although special, this simplifies the subsequent algebra and our later results are not sensitive to this precise assumption. In this case, (29) implies simply that:

$$V + D = 0. \tag{30}$$

In other words, aggregate domestic demand for financial assets is zero, in the steady state. It then follows that an increase in government debt (an exogenous variable, here) 'crowds out' net foreign assets one-for-one in the long run.[17]

Let us sum up the steady-state response to an increase in government debt. A rise in D, by crowding out net foreign assets, V, reduces aggregate consumption, C. This linkage occurs through (28) and (30). The mechanism is that, with a fall in V, interest receipts from abroad decline, obliging domestic households to cut back their consumption. At the same time, being poorer, they also cut back on leisure and so increase labour supply, thereby boosting domestic

[17] The same result arises in Blanchard (1985) in the small open economy case. More generally, both here and there, if the foreign real interest rate is above (below) the domestic time preference rate, then an increase in government debt crowds out steady-state net foreign assets by more (less) than 100%.

output. The greater availability of domestic goods on world markets then lowers their price, so that the country's terms of trade worsen. These linkages to Y and ρ occur through (27).[18]

3.2 Log-linearisation, reduction and separation of the model

The macroeconomic structural equations contain significant non-linearities. Although the full non-linear version of the model can be studied using numerical simulations, if we wish to directly inspect the linkages between macroeconomic variables using algebra then it is essential, for tractability, to take a log-linear approximation of the model.[19] The non-linearity also means that the log-linearised equations are sensitive to the steady state around which the approximation is taken. Here we choose the steady state with zero inflation, zero government debt and zero government spending (the 'reference' steady state). Note that $D = 0$ implies $V = 0$ in the reference steady state (from (30)), while $V = 0$ means that $B = 0$ (from $B + r^F V = 0$).

Once the raw equations have been log-linearised, the mathematical logic of the model is best revealed by reducing the resulting system of equations to one with a simpler structure. The details of this reduction are given in Appendix B. Its outcome is (31)-(35) below. Lower-case letters in what follows generally denote log-deviations, i.e. $z_t \equiv \ln Z_t - \ln Z_R$ for any variable Z_t, where Z_R is its value in the reference steady state.[20]

$$v_{t+1} = (1+r^F)v_t + b_t, \tag{31}$$

$$b_{t+1} = b_t + \zeta(1-\gamma)(v_{t+1} + d_{t+1}), \tag{32}$$

$$\pi_t^H = (1+r^F)^{-1}\pi_{t+1}^H + \kappa\tilde{y}_t, \qquad (\pi_t^H \equiv p_t^H - p_{t-1}^H; \quad \tilde{y}_t \equiv y_t - y_t^N) \tag{33}$$

$$\tilde{y}_t = \hat{\rho}_t - [\gamma(1-\gamma)^{-1} + \delta]b_t + (1-\delta)g_t^H, \qquad (\hat{\rho}_t \equiv -p_t^H) \tag{34}$$

$$y_t^N = \delta(b_t + g_t^H). \tag{35}$$

[18] These results are qualitatively unaffected by assuming r^F is in some region either side of $1/\beta$ - 1.
[19] Log-linearisation of models with Calvo-style price staggering also eliminates some potentially interesting features of the dynamics, as Ascari (2004) shows. Nevertheless it is a natural first step towards obtaining a deeper understanding of the model's characteristics.
[20] Further details of variables' definitions are given in Appendix B.

Here (ζ, κ, δ) are composite parameters whose definitions are given in Appendix B. They are all positive, but it should be noted that ζ tends to zero as q tends to one, and that $\delta < 1$.

A key feature of this semi-reduced form of the model is that it separates into two sub-systems. The first sub-system consists of (31) and (32), which gives the dynamics of net foreign assets and of the trade balance. (31) is just the log-linearised version of the balance of payments equation, (24). Note that v_t, whose evolution (31) governs, is a naturally predetermined variable. (32) is obtained by combining the net export demand function, the consumption Euler equation and the real UIP condition. Note that b_t, whose evolution (32) governs, is a naturally non-predetermined variable. Together, (31) and (32) form a self-contained second-order difference equation system. Given a path for the exogenous government debt, d_{t+1}, they can be used to find the perfect-foresight paths of (b_t, v_t) without reference to the rest of the economy.

The second sub-system consists of (33) and (34), which gives the dynamics of inflation and the output gap. (33) is a typical New Keynesian Phillips Curve (NKPC) equation, governing the evolution of π_t^H. It is obtained, inter alia, by combining the price-setting equation with the price index equation and imposing labour market clearing. (34) is an 'aggregate demand' function, albeit of an unfamiliar type. It provides a contemporaneous positive linkage of the output gap to the terms of trade variable, but one into which the trade balance and government spending also enter. (34) is essentially the inverted net export demand function, but from which total domestic consumption has been solved out, being replaced by the output gap. Noting that the terms of trade variable just equals minus the domestic price level, it can be seen that (33) and (34) together constitute an implicit second-order difference equation in p_t^H, but with b_t as a forcing variable. Hence, having solved the first sub-system for the time path of b_t, we can proceed to use this in the second sub-system to solve for the time paths of (p_t^H, $\tilde{y}_t^H$). In this way the solution of what is, overall, a fourth-order dynamical system is greatly simplified, making the resulting mathematical expressions amenable to direct economic interpretation.

In this model, the output gap, $\tilde{y}_t$ ($\equiv y_t - y_t^N$) behaves significantly differently from output itself. This is because the 'natural' level of output, y_t^N (defined as the value which y_t would take if prices were perfectly flexible), is also affected by government debt, government spending and net foreign assets, as they change over time. This occurs through the influence of these variables on labour supply. (35) shows that movements in y_t^N in fact follow movements in the trade balance and government spending. Although such dependence on b_t may seem unintuitive, note that b_t is a function of net foreign assets along the perfect-foresight path, so that y_t^N can equivalently be viewed as a function of v_t. The details of this are discussed further below. Having solved for $\tilde{y}_t$ and y_t^N, we can then, if wished, recover the solution for total output, y_t, which is just their sum.

4. Effects of a One-Period Budget Deficit

As stated earlier, for our main fiscal policy experiment we consider a one-period budget deficit, implemented through a cut in the level of lump-sum taxation, τ_t, and financed by issuing government debt. After the impact period, taxation is raised again in order to balance the budget, so that government debt (d_{t+1}) is thereafter constant at the new, higher, level. Although more 'realistic' policy changes could be examined, we choose this one in order to reveal as clearly as possible the dynamic forces which are unleashed. Under this experiment, government debt does not continuously evolve, so the subsequent time path of the economy is driven only by the responses emanating from the domestic and foreign private sectors, and not by the dynamics of the policy instrument itself.

4.1 Effects on the trade balance, net foreign assets and the natural level of output

As just seen, the time paths of (b_t,v_t) are determined only by (31) and (32). Here, v_t is predetermined and b_t is non-predetermined. Therefore, for a unique bounded perfect-foresight equilibrium to exist, we need this second-order system to have one eigenvalue inside, and one outside, the unit circle. Appendix C demonstrates that this is always satisfied.

We depict the equilibrium in Figure 1. The stationary loci are given by:

$$\Delta b_{t+1} = 0: \quad b_t = -(1+r^F)v_t - d \,, \tag{36}$$

$$\Delta v_{t+1} = 0: \quad b_t = -r^F v_t \,. \tag{37}$$

Initially, with $d = 0$, the economy is in a steady state at the origin. When d is permanently increased[21], the $\Delta b_{t+1} = 0$ locus shifts down. The new steady state is at point S, to which there is a unique convergent path – the saddlepath – given by the dashed line. On impact, the economy jumps onto this path at point A and the trade balance thus goes into deficit. Over time, the economy then converges to point S along the saddlepath. Hence, during the transition, domestic households steadily accumulate foreign debt, while the trade balance gradually turns from deficit to surplus. In the new steady state, the surplus is just enough to offset the continuous outflow of interest payments on the permanently higher foreign debt.

Algebraically, the solutions for v_t and b_t, following a tax cut in $t = 0$, can be expressed as:

$$v_t - v \;=\; \lambda_1^t [v_0 - v], \qquad\qquad b_t - b \;=\; (\lambda_1 - 1 - r^F)[v_t - v], \tag{38}$$

where v is the new steady-state value of v_t (= $-d$), v_0 is the pre-shock steady-state value (= 0), b is the new steady-state value of b_t (=$r^F d$), and λ_1 is the stable eigenvalue of the system. We can show (see Appendix C) that λ_1 lies, more specifically, in the interval (0,1).[22]

It follows from the behaviour just described for b_t, and (35), that the natural level of output (y_t^N) also falls on impact. Along the transition path, however, y_t^N rises, ending up higher than its pre-shock level. This is directly illustrated in panel (a) of Figure 2. We can understand these effects as resulting from labour supply changes. The increase in government debt initially raises domestic households' perceived total lifetime wealth. This is due to $q < 1$, which causes the increased current bond holdings not to be fully offset by the expected higher present value of the taxes during the lifetime of currently alive households. Such, of course, is the standard mechanism breaking Ricardian Equivalence under OLGs. Feeling wealthier, households demand more leisure and supply less labour. Under fully flexible prices, this has a negative supply-side effect on output. Over time, however, households' wealth declines due to

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[21] From the linearised government budget constraint, $g_t^H - \hat{\tau}_t + (1 + r^F)d_t = d_{t+1}$ (where $g_t^H = 0$ for this experiment), we can see that the increase in d_{t+1} corresponds to a one-off cut in $\hat{\tau}_t$, while in all subsequent periods $\hat{\tau}_t$ will be higher by $r^F d$.

[22] Note that the foregoing only applies when $q < 1$ (and thus $\zeta > 0$), i.e. when Ricardian Equivalence is absent. When $q = 1$, the system exhibits path-dependence. In this case, $b_t = -r^F v_0$, where v_0 is the predetermined initial stock of net foreign assets. b_t and v_t are then time-invariant along the perfect-foresight path and the system is independent of d.

the accumulation of net foreign debt. In the new steady state the country is poorer than when it started, since it has to make permanently higher interest payments to its foreign creditors. This has the opposite effect to the initial shock, with domestic households now demanding less leisure and supplying more labour, implying an expansionary long-run effect on the natural level of output.

4.2 Impact effects on the output gap, output and the price

As explained in 3.2, the sub-system (33)-(34) which describes the dynamics of $(\pi_t^H, \tilde{y}_t)$ amounts to an implicit second-order difference equation in p_t^H, but one in which b_t enters as a forcing variable. We can now use the solution just obtained for b_t, (38), to re-write this as a self-contained third-order system in (p_t^H, p_t^{HP}, v_t), where v_t and p_t^{HP} $(\equiv p_{t-1}^H)$[23] are predetermined variables while p_t^H is non-predetermined. Appendix D provides details. Although third-order, this transformed system can easily be studied analytically because v_t evolves independently of (p_t^H, p_t^{HP}). For a unique bounded perfect-foresight solution to exist, we need it to possess two eigenvalues inside, and one outside, the unit circle. In Appendix D we show that this condition is indeed satisfied. One of the stable eigenvalues is λ_1, inherited from the (b_t, v_t) sub-system. The other we will call λ_2. Appendix D shows that it, too, lies more specifically in the interval (0,1).

We now seek to discover the immediate effect which the budget deficit has on the output gap, i.e. on $\tilde{y}_0$. Since the economy is assumed to start in the 'reference' steady state, the pre-shock values of all deviation variables are zero. The solution which we obtain for $\tilde{y}_0$ is:

$$\tilde{y}_0 = \frac{\lambda_2(1-\lambda_1)[1+r^F-\lambda_1+(1-\lambda_2)(1+r^F)]}{1+r^F-\lambda_1\lambda_2}[\gamma(1-\gamma)^{-1}+\delta]d \tag{39}$$

The derivation is spelled out in Appendix D. In this log-linearised model, the coefficient on d is the 'impact multiplier'. It is clear from (39) that this is positive. Thus it is unambiguous that the fiscal deficit causes a boom, in the sense of a positive output gap. This is what we would intuitively expect: the higher government debt adds to the lifetime net wealth of domestic

[23] As well as being the lagged value of p_t^H, the variable p_t^{HP} can also be interpreted as the index of all prices, $p_t(z)$, which are still in force in period t but which are 'predetermined', i.e. which were set in period t-1 or earlier.

households, and in the presence of sticky prices the stimulus which this gives to consumption demand pushes output above its natural level. Unsurprisingly, then, given the Keynesian elements of staggered prices and OLGs, the economy's short-run behaviour is typically Keynesian.

As can be seen from (39), the magnitude of the multiplier is particularly sensitive to the magnitudes of the two stable eigenvalues, λ_1 and λ_2. In Appendix C we show that the most important parameter determining λ_1 is q; and in Appendix D we similarly show that the most important parameter determining λ_2 is α. As q goes from 0 to 1, λ_1 goes monotonically from 0 to 1; and as α goes from 0 to 1, λ_2 likewise goes monotonically from 0 to 1. It follows that if agents were infinitely-lived (q = 1), or if prices were fully flexible (α = 0), then the impact effect on the output gap would be zero. As 1-q and α move away from zero, i.e. as expected lifetimes fall and price stickiness rises, the size of the impact effect on the output gap increases. Although the set-up of the model would probably have led us to predict such results, here they are visible directly from the algebra of the solution in a clean and transparent way.

One might also be interested in the impact effect on the absolute level of output, and not just on its value relative to its natural level. We saw that the natural level of output falls on impact. Since $y_t = \tilde{y}_t + y_t^N$, it follows from the foregoing that, for sufficiently flexible prices, the rise in the output gap will be too small to outweigh the fall in the natural level of output, and thus absolute output will fall. On the other hand, we can show that for sufficiently sticky prices, y_t will certainly rise. The reason why an expansionary impact on output itself is not guaranteed is that the fiscal deficit has a contractionary supply-side effect, via labour supply, which counteracts the expansionary demand-side effect, via consumption demand.

We encapsulate these findings regarding the impact effects on output as:

Proposition 1. *The impact effect on the output gap of a one-period debt-financed tax cut is positive, except when either the probability of being unable to change price (α), or the probability of dying (1-q), is zero. It is increasing in α and 1-q. The impact effect on output itself is positive if α is sufficiently close to one, but negative if α is sufficiently close to zero.*

As regards the impact effect on the price level, p_0^H, we can readily show that it is always positive except when agents are infinitely-lived (q = 1), the latter case making it zero.

4.3 Behaviour of the output gap and inflation along the transition path

How do the output gap and inflation evolve during the transition to the new steady state? We know they must eventually tend to zero. However, since the dynamics along the perfect-foresight path are governed here by two stable eigenvalues, rather than just one, it is possible that this adjustment is non-monotonic, even though both eigenvalues are real and lie in the (0,1) interval.

To examine this, we derive the 'final form' solutions for the variables of interest, expressing them as explicit functions of time. In the case of the output gap, the equation obtained is (see Appendix E for the derivation):

$$\tilde{y}_t \;=\; [w'_{y1}\lambda_1^t \;+\; w'_{y2}\lambda_2^t][\gamma(1-\gamma)^{-1}+\delta]d\,, \tag{40}$$

where

$$w'_{y1} \;=\; \frac{\lambda_2(1-\lambda_1)(1+r^F-\lambda_1)^2}{(\lambda_2-\lambda_1)(1+r^F-\lambda_1\lambda_2)},$$

$$w'_{y2} \;=\; \frac{\lambda_2(1-\lambda_1)[r^F(1+r^F-\lambda_1\lambda_2)+(1+r^F)(1-\lambda_2)^2]}{(\lambda_1-\lambda_2)(1+r^F-\lambda_1\lambda_2)}.$$

Now consider the signs of the coefficients (w'_{y1}, w'_{y2}). Both have positive numerators, but their denominators depend on the sign of λ_2-λ_1. If $\lambda_2 > \lambda_1$, then w'_{y1} is positive and w'_{y2} is negative; while if $\lambda_2 < \lambda_1$, their signs are reversed. Both $\lambda_2 > \lambda_1$ and $\lambda_2 < \lambda_1$ are possible within our assumptions.

Suppose first that $\lambda_2 > \lambda_1$, implying $w'_{y1} > 0$, $w'_{y2} < 0$. Slightly rearranging (40) gives:

$$\tilde{y}_t \;=\; \lambda_2^t\{w'_{y1}(\lambda_1/\lambda_2)^t \;+\; w'_{y2}\}[\gamma(1-\gamma)^{-1}+\delta]d\,. \tag{41}$$

The sign of the term {.} at first appears ambiguous. However, we already know that $\tilde{y}_t$ is positive when t = 0, from which it follows that $w'_{y1}+w'_{y2} > 0$. In other words, when t = 0 the positive term involving w'_{y1} inside {.} must dominate the negative term w'_{y2}. Since, by assumption, $\lambda_1/\lambda_2 < 1$, it is then clear that, over time, the size of the positive term shrinks towards zero, so that the sign of $\{.\}$ must eventually switch from positive to negative. Thus the output gap, having started out positive, must at some point become negative. The initial 'boom' inevitably turns into a 'bust'. It is also clear that this change of sign can occur only once. The boom-bust cycle is thus not repeated: output will subsequently tend to zero, but always from below.

Next suppose that $\lambda_2 < \lambda_1$. Since the signs of (w'_{y1}, w'_{y2}) are then reversed, an exactly parallel argument, but this time factorising (40) by λ_1^t, shows that the output gap must again exhibit a boom-bust cycle.

A third possibility is $\lambda_1 = \lambda_2$. This could only arise for a chance combination of the parameter values. We can show that, in this case too, a boom-bust cycle must occur.[24]

Thus we have established our main result:

Proposition 2. *Although the output gap is positive on impact, it must later become negative. It then tends to zero asymptotically from below. This holds for all parameter values (other than the special cases associated with a zero impact effect).*

The general shape of the time path for the output gap is hence as depicted in panel (b) of Figure 2. Below, we discuss what are the macroeconomic forces which result in this switch from boom to bust.

Next consider the transition path of inflation, π_t^H. Using similar methods we can show that, having started out positive, inflation must at some point switch to being negative, and cannot change sign thereafter. In fact, overall, inflation must be more negative than positive. This last follows from our earlier finding that in the new steady state the terms of trade must have worsened, which means that the domestic price level (p^H) must end up below its pre-shock value. The fact that the country is part of a common currency area is key to this. If it had its own currency, the terms of trade deterioration could be achieved through exchange rate depreciation. However, without the possibility of exchange rate adjustment, a fall in the

[24] The calculations for this case are available on request. Since there are now 'repeated' eigenvalues, the solution (42) needs to be modified.

domestic price level is the only way to ensure this outcome. The main features of inflation's behaviour can thus be summed up as:

Proposition 3. *Inflation becomes positive on impact but later turns negative. It then tends asymptotically to zero from below. On average, over the whole transition path, inflation is negative. This holds for all parameter values (other than the special cases associated with a zero impact effect).*

A typical time path for inflation is illustrated in panel (d) of Figure 2.

Both the output gap and the inflation rate therefore go through their own boom-bust cycles. It is notable that, as illustrated in Figure 2, inflation is very likely to go negative while the output gap is still positive. This is because, if we treat r^F as negligible, the NKPC equation, (34), can be rewritten as $\tilde{y}_t = (\pi_t^H - \pi_{t+1}^H)/\kappa$, showing that the sign-change point for the output gap is where inflation reaches a turning point. In the present case, this is where inflation is at a minimum, and thus where it has already become negative.

4.4 The effects of a balanced-budget increase in government spending and a comparison with the effects of a fiscal deficit

To understand why a fiscal-deficit-induced boom inevitably turns into a bust, as opposed to just fading away, it is helpful to consider instead what happens under a different type of fiscal stimulus policy: an increase in government spending which is tax-financed. Such a 'balanced-budget' expansion has been much re-examined in New Keynesian models in recent years so we shall not present a full analysis here[25], but it is instructive to review how it would perform in our small open economy model under a common currency. The relevant log-linearised model remains that which is described by (31)-(35) above. We now suppose that in some period g_t^H is raised permanently from zero to a positive value, g^H, and that this is financed by an equal rise in $\hat{\tau}_t$ so that government debt, d_t, stays at zero.

It is clear that the change in g_t^H does not disturb the (b_t,v_t) sub-system, (31)-(32). The trade balance and net foreign assets are hence unaffected. Assuming the economy was initially

[25] See Woodford (2011) for a survey.

in the reference steady state, b_t and v_t would remain at zero. The OLG structure therefore plays no role in the case of a balanced-budget spending change, because the effects of OLGs (i.e. of $q < 1$) are channelled through the (b_t, v_t) sub-system. The macroeconomic outcomes would be the same even if agents were infinitely-lived. The increase in government spending does, however, raise the natural level of output, as can be seen from (35). This is because the higher tax burden generated by the higher spending causes households to reduce their demand for leisure, and so to supply more labour. Such an expansionary supply-side effect, working through an income effect on labour supply, is a familiar feature of many related models.

Turning to the 'residual' equation sub-system, (33)-(34), the fact that b_t is unaffected by government spending (and should thus be set to zero) means that (33)-(34) can be used just by itself to determine inflation and the output gap. (33), the NKPC equation, is an 'aggregate supply' relation; while (34) is an 'aggregate demand' relation, and it is through the latter that government spending affects the system. From what has been said, we can rewrite (35) as:

$$\tilde{y}_t = (1-\delta)g_t^H - p_t^H, \tag{42}$$

where we recall that $\delta < 1$ (see Appendix B). Here we have a simple negative relationship between $\tilde{y}_t$ and p_t^H. It shows that an increase in g_t^H will raise $\tilde{y}_t$ unless it is offset by an increase in p_t^H.

It is a straightforward exercise to solve this sub-system to demonstrate the effects of a permanent increase in g_t^H. (33)-(34) can be written as a second-order difference equation in p_t^H, or as a pair of first-order equations in (p_t^H, p_t^{HP}) (cf. sub-section 4.2). For a unique bounded solution we need it to have a single stable eigenvalue. It can be verified that it indeed possesses such an eigenvalue, which is moreover the same as λ_2 above. In this way the solution for $\tilde{y}_t$ (for $t \geq 0$) is found to be:

$$\tilde{y}_t = \lambda_2^{t+1}(1-\delta)g^H. \tag{43}$$

This shows that a balanced-budget government spending increase produces a boom which fades away monotonically, in contrast to a debt-financed tax cut which produces a boom which turns to bust before fading away. Monotonic convergence following a balanced-budget spending increase is the standard outcome in a closed economy, so it is not particularly surprising to find that this generalises to a small open economy.

Now return to the case of a tax cut. As seen in sub-section 4.2, the equation system governing the dynamics of inflation and the output gap is again (33)-(34), with the only difference being that the 'driver' of aggregate demand in (34) is not g_t^H, but b_t. To make this more explicit, re-write (34) as:

$$\tilde{y}_t \;=\; [\gamma(1-\gamma)^{-1}+\delta](-b_t) \;-\; p_t^H . \tag{44}$$

Comparing (44) with (42), we note that it is as if g_t^H has been replaced by $-b_t$ (in either case multiplied by a positive coefficient) as the source of shifts in the aggregate demand relation. The trade deficit ($-b_t$) is not a variable which we normally think of as a source of shifts in aggregate demand. However, we may notice that $-b_t$ is proportional to the composite variable $c_t-\gamma\hat{\rho}_t$ (as can be seen from the log-linearised net export demand function, (B2), in Appendix B). In turn, $c_t-\gamma\hat{\rho}_t$ can be interpreted as total domestic composite consumption but measured in foreign goods units, i.e. as $P_tC_t\,/\,P_t^F$ (after expressing it in deviation form). This is made apparent by writing $P_tC_t\,/\,P_t^F$ as $C_t\,/\,(\tilde{\gamma}\rho_t^{\gamma})$. Therefore if we define $c_t-\gamma\hat{\rho}_t$ as c_t^W (total domestic composition consumption in 'world' goods units), (44) equivalently becomes:

$$\tilde{y}_t \;=\; [\gamma+(1-\gamma)\delta]c_t^W \;-\; p_t^H , \tag{45}$$

where $c_t^W=(1-\gamma)^{-1}(-b_t)$ (by virtue of (B2)).

Viewed in terms of (45), the driver of aggregate demand in the case of a debt-financed tax cut is c_t^W. c_t^W follows a time path which is the mirror image of that of b_t (or of y_t^N, which is shown in panel (a) of Figure 2). Its evolution, like that of b_t, is independent of the sticky-price part of the economy. It is depicted in panel (c) of Figure 2. The increase in government debt at first stimulates demand as measured by c_t^W, but over time this stimulus declines monotonically as the country decumulates net foreign assets, and it finishes by turning into a permanent drag on demand. Relative to the case of a balanced-budget increase in government spending, therefore, the stimulus to demand provided by a fiscal deficit is temporary. This is true even though the increase in government debt itself is permanent.

The temporary nature of the stimulus to demand, in turn, is what explains why the boom turns to bust. From (45) we see that, to keep the output gap at zero, the domestic price level would need to track the movements of c_t^W (times its associated coefficient) exactly: it would need to jump up on impact, and then gradually fall back. However, with staggered price setting, this is not possible. Instead what happens is that p_t^H lags behind c_t^W (times its coefficient): p_t^H starts to rise, but after a certain interval c_t^W has become negative, so p_t^H now needs to fall. Having started out too low, at a certain point the price level overshoots and becomes too high. At this point the boom turns to bust. This is also illustrated in panel (c) of Figure 2, where the time path of p_t^H is superimposed on that of c_t^W (times its coefficient). Where they cross is where the output gap goes from positive to negative. By contrast, in the case of a balanced-budget increase in government spending, the reason a bust does not occur is that the price level never needs to fall. The stimulus to aggregate demand in this case is permanent. While price stickiness again causes an output gap to emerge, the price level is always too low relative to the source of the aggregate demand shift, and so it always catches up with this shift 'from below'.[26]

4.5 A numerical example

Our model is designed to facilitate a qualitative understanding of the mechanisms at work, rather than to produce realistic quantitative estimates. Nevertheless one might be interested to know the rough magnitudes that even such a skeletal apparatus can yield, for plausible parameter values. We here present a numerical example which provides an idea of this. Although we illustrate only one case, we report how varying some of the parameter values alters the outcomes. The time paths of the main variables of interest can easily be computed using the explicit algebraic solutions obtained earlier. They are graphed in Figure 3.

The parameter values used are given in the caption to Figure 3. One time period is taken to be a quarter and the deficit is set at 1% of pre-shock GDP. The value used for q implies an

[26] An alternative explanation for the bust which might initially seem tempting is to observe that taxes are first cut but then raised again, so the boom-then-bust in the output gap might seem just to reflect this down-then-up pattern in taxation. However this is not a satisfactory explanation, because households' behaviour is based on intertemporal optimisation under perfect capital markets, so that it is the whole expected future time path of taxation, not just current taxation, which affects this behaviour.

expected remaining lifetime (1/([1-q]) of 5 years. This choice is guided by the consideration that, in the uncertain lifetimes model, it is well known that if the expected remaining lifetime is calibrated to a typical human lifetime, then only a trivial degree of departure from Ricardian Equivalence is implied. It is hence better to interpret 'death' as representing a broader set of events than just physical death. The interesting study by Bayoumi and Sgherri (2006) econometrically estimates q based on US data on consumption, income and taxation, and finds that an expected 'economic lifetime' of about 5 years provides a good fit. Of the other parameter values, α is chosen such that the expected duration of a 'price spell' (1/[1-α]) is one year. ψ is given a value to make the fraction of a household's time devoted to leisure equal to 1/3, in the reference steady state.

Regarding the relative magnitudes of the impact effects exhibited in Figure 3, it is notable that the increase in the output gap dominates the decrease in natural output, such that output itself (which is the simple unweighted sum of the two) clearly increases. This is what we would expect: in the short run, the demand-driven, expansionary effect strongly outweighs the supply-driven contractionary effect. Nevertheless the latter is not completely trivial. A lower value of ψ reduces its importance, by weakening the income effect on labour supply.

The dynamic effects seen in Figure 3 are strongly persistent. This reflects the fact that both eigenvalues turn out to be high, with $\lambda_1 = 0.9843$ and $\lambda_2 = 0.8872$. As already noted, q and α (respectively) are the main determinants of these. For plausible values of q, a near-unit value of λ_1 is always likely, because a 'biologically realistic' q would raise λ_1 still further. If such a value of q is used instead, its main consequence is to scale down the absolute magnitudes of all the effects, but not to change the shapes of time paths markedly. As regards λ_2, this is lower if α is lower; or if θ is lower; or if σ is higher. However λ_2 is unlikely to fall below about 0.75, for plausible parameter choices.

The point of greatest interest is to observe the 'bust' relative to the 'boom'. The plot of the output gap in Figure 3 shows that the bust is shallow relative to the boom. Measuring each at its maximum extent, the former is 3.8% of the latter. On the other hand, we can also see that the bust is prolonged. If we consider the *cumulative* loss of output as measured by the output gap, in the bust phase we find that it is 33.1% of the cumulative gain of output in the boom phase.[27] Viewed this way, the bust is of significant magnitude relative to the boom. Other numerical experiments confirm that, for parameter values in an empirically relevant range, the bust is likely to be shallow but long. The parameter to which it is most sensitive is r^F, the foreign real interest rate. With a lower value of r^F, such as 0.01, the bust becomes relatively insignificant, unless other parameters such as α are pushed to fairly extreme values. Both 0.01 and 0.02, however, lie in the range of international borrowing and asset return rates which are broadly realistic. This therefore implies that the possibility that a fiscal deficit will be the source of a sizeable delayed recession is real.

5. Conclusions

We have studied in depth the mechanisms which operate following a one-period, debt-financed, fiscal deficit in a small open economy. The economy incorporates New Keynesian features which capture effects which are present in simple ad hoc Keynesian models, in particular price stickiness and non-neutrality of public debt, but in a way consistent with general equilibrium principles. To do this it assumes monopolistic competition combined with Calvo-style staggered price setting, and overlapping generations based on uncertain lifetimes. Since our objective is not the quantitative one of accurately replicating a real-world data set, but the analytical one of being able to clearly observe the inner workings of the economy, we have eliminated many inessential, but undeniably 'realistic', features which would otherwise complicate the picture. However, the resulting dynamical system is still non-trivial, exhibiting fourth-order dynamics.

Notwithstanding the order of the dynamics, we have been able to fully characterise the dynamic responses of variables to the deficit algebraically, by exploiting a separability property of the system. This consists in the fact that the time paths of net foreign assets and the trade balance can be solved for independently of other variables. Taking advantage of this, we were able to show that, on impact – as might be expected from the Keynesian structural characteristics of the economy – the fiscal deficit reliably causes a boom, i.e. a positive output

[27] This figure is reached by truncating the bust at 120 periods.

gap. However we also found that the boom at some point inevitably turns into a bust, i.e. a negative output gap. This second finding is not a well-known consequence of a fiscal deficit, and yet it follows robustly from the same New Keynesian structural features which give rise to the initial boom. The wider implications of our analysis are that, while fiscal deficits can indeed be used as a tool for business cycle stabilisation, they should not be used in a naive way. A simple one-period deficit sets off a dynamic reaction which can itself be destabilising. A careful plan for the future path of the fiscal deficit and public debt levels therefore needs to be mapped out which can mitigate the disturbance exhibited in the dynamic reaction. To do this is beyond the scope of the present work, but it would seem a worthwhile topic for future investigation.

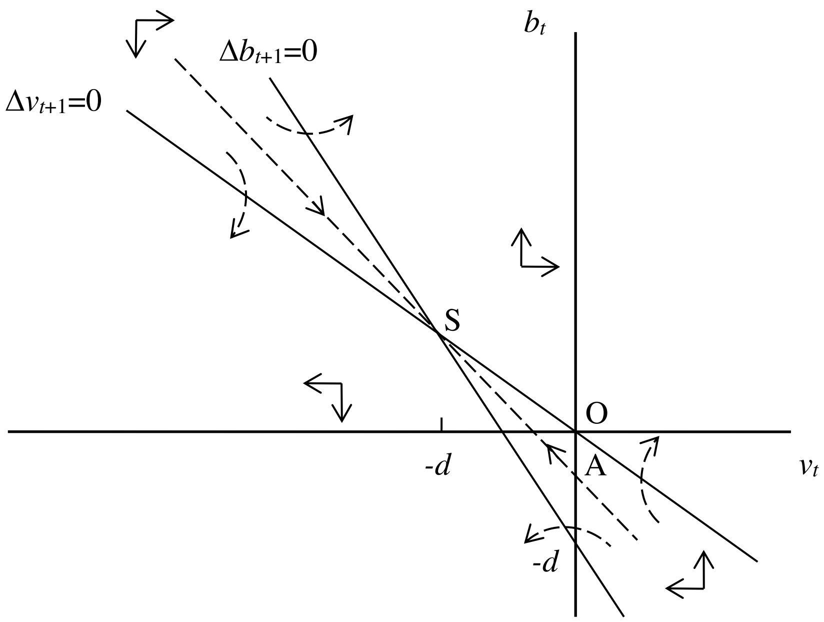

Figure 1 The effects of a one-period fiscal deficit on the trade balance and net foreign assets

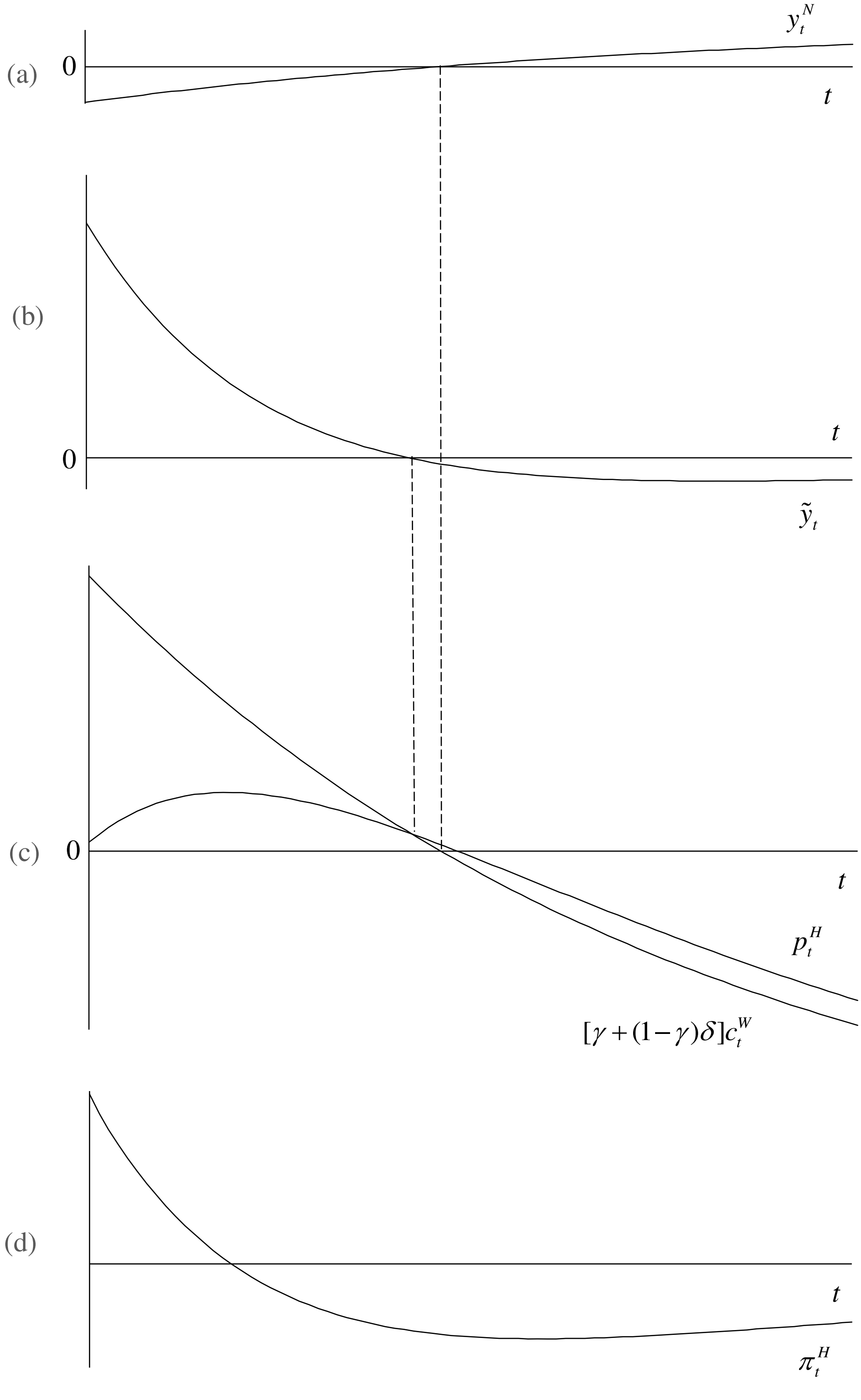

Figure 2 Qualitative illustrations of the time paths of variables following a one-period fiscal deficit

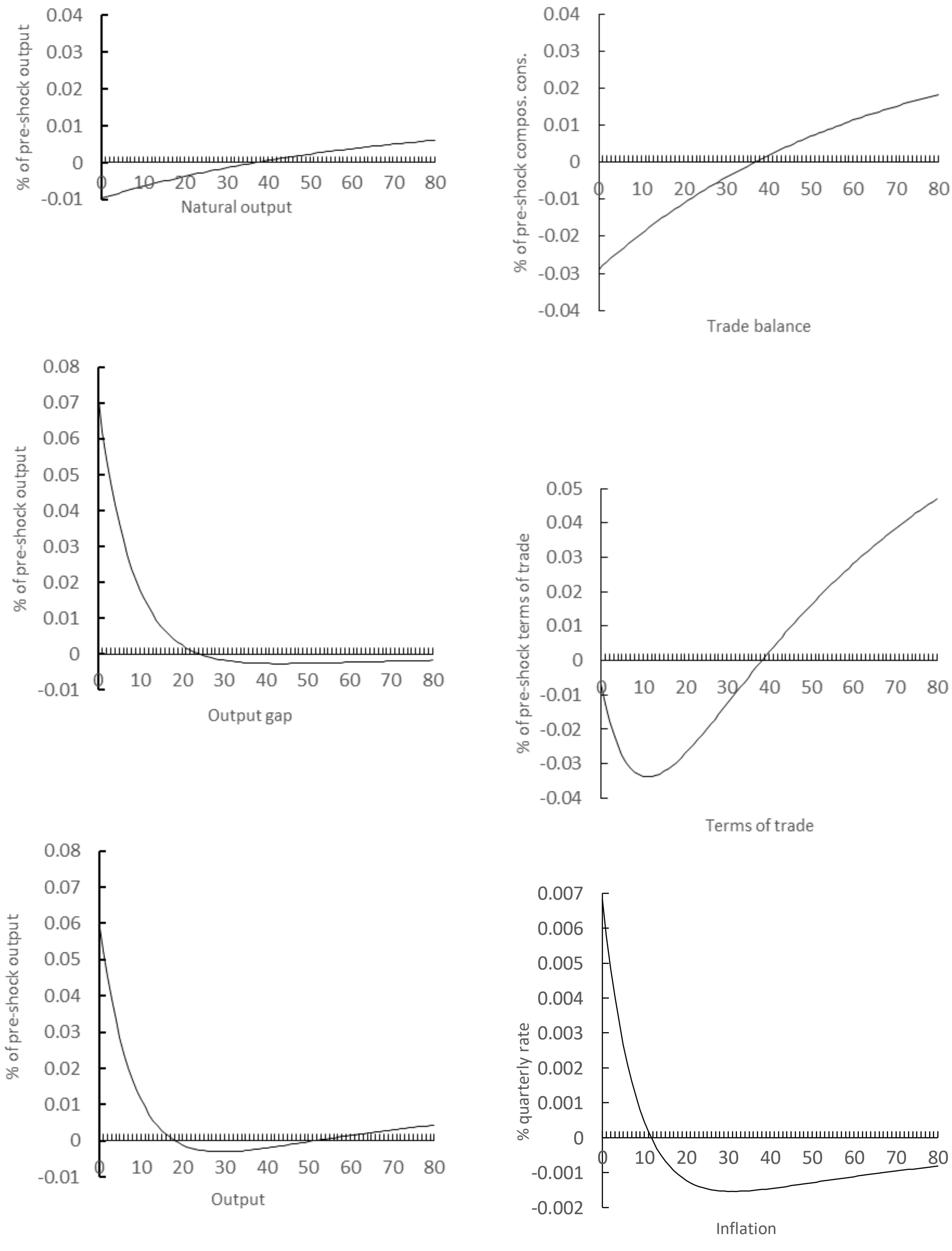

Figure 3 A numerical simulation of selected variables' time paths in response to a one-period fiscal deficit equal to 1% of pre-shock output (q = 0.95, α = 0.75, r^F = 0.02, θ = 15, γ = 0.7, σ = 0.5, ψ = 0.9333, K = such that ρ_R = 1, 1 period = 1 quarter).

Appendix A

In a zero-inflation steady state, symmetry amongst all domestic firms implies that the prices of all goods varieties will be the same, i.e. $P(z) = P^H = X$ for all z. The price-setting equation, (12), then reduces to $W/P^H = (1\text{-}1/\theta)\sigma Y^{1\text{-}1/\sigma}$. This is essentially the economy's labour demand function, although expressed in terms of output rather than employment. Combining it with the labour supply function, (7) (and substituting out P^H/P using (16)), we obtain:

$$Y^{1/\sigma} = 1 - \frac{\psi}{\tilde{\gamma}\rho^{\gamma-1}}\frac{\theta}{\theta-1}\frac{1}{\sigma}Y^{1/\sigma-1}C\,, \tag{A1}$$

which is the steady-state labour-market clearing condition.

(A1) provides a negative relationship between Y and ρ for a given C. A second relationship between Y and ρ for a given C – this time positive – is given by the world demand function for home-produced goods, (20). (G^H is also a parameter in this function.) When solved simultaneously, they determine Y and ρ as implicit functions of C and G^H. These are the functions represented as (27) in the main text.

To show why the functions (27) are both decreasing in C, we may use (A1) to eliminate ρ from (20). This gives:

$$Y = \frac{\sigma\gamma(\theta-1)}{\psi\theta}\frac{1-Y^{1/\sigma}}{Y^{1/\sigma-1}} + K\left[\frac{\sigma\tilde{\gamma}(\theta-1)}{\psi\theta}\frac{1-Y^{1/\sigma}}{CY^{1/\sigma-1}}\right]^{1/(1-\gamma)} + G^H\,. \tag{A2}$$

Notice that an increase in Y unambiguously raises the LHS and lowers the RHS of this equation, while an increase in C unambiguously lowers the RHS. Hence it implies an unambiguously negative relationship between Y and C. To see the sign of the relationship between ρ and C, return to look at (20). The LHS is clearly increasing in Y, while the RHS is unambiguously increasing in C and in ρ. Having just seen that a rise in C lowers Y, it then follows that, when C increases, a fall in ρ is necessary to maintain equality between the sides. Hence the relationship between ρ and C is also unambiguously negative.

Appendix B

A set of 'raw' log-linearised equations is given in (B1)-(B6) below. As stated in the main text, lower-case letters generally denote log-deviations, i.e. $z_t \equiv \ln Z_t - \ln Z_R$ for any variable Z_t, where Z_R is its value in the 'reference' steady state. Where the variable is already in the lower case, $\hat{}$ is used. In the case of interest rates, log-deviations are taken of 'gross' values, so $\hat{r}_t \equiv \ln(1+r_t) - \ln(1+r_R)$, etc. Where the reference steady state value is zero, so its log is not defined, the 'deviation' form of the variable is obtained by scaling it by an appropriate reference steady state value: hence $b_t \equiv B_t / C_R$, $v_t \equiv V_t / C_R$, $d_t \equiv D_t / C_R$, $g_t^H \equiv G_t^H / Y_R$, $\hat{\tau}_t \equiv T_t / C_R$. Inflation rates are defined directly as $\pi_t \equiv p_t - p_{t-1}$, $\pi_t^H \equiv p_t^H - p_{t-1}^H$. It is also helpful to define reference steady state values of the nominal 'scale' variables, (P_t, P_t^H, P_t^F). We let $P_R \equiv 1$, so (5) then implies $P_R^H \equiv \tilde{\gamma}\rho_R^{\gamma-1}$ and $P_R^F \equiv \tilde{\gamma}\rho_R^{\gamma}$, where ρ_R is tied down by the steady state equations.

$$y_t \;=\; \gamma c_t \;+\; (1-\gamma^2)\hat{\rho}_t \;+\; g_t^H \qquad [\,\hat{\rho}_t \;=\; -\,p_t^H\,] \tag{B1}$$

$$b_t \;=\; (1-\gamma)(\gamma\hat{\rho}_t - c_t) \tag{B2}$$

$$c_t \;=\; c_{t+1} \;-\; \hat{r}_{t+1} \;+\; \zeta(v_{t+1} + d_{t+1}) \qquad [\,\zeta \equiv (1/q-1)(1+r^F-q)(1+\psi)^{-1}\,] \tag{B3}$$

$$\hat{r}_{t+1} \;=\; \gamma(\hat{\rho}_{t+1} - \hat{\rho}_t) \tag{B4}$$

$$v_{t+1} \;=\; (1+r^F)v_t \;+\; b_t \tag{B5}$$

$$\pi_t^H \;=\; (1+r^F)^{-1}\pi_{t+1}^H \;+\; \kappa_a\{[1/\sigma - 1 + (\theta-1)/\theta\psi]y_t \;+\; c_t \;+\; (1-\gamma)\hat{\rho}_t\} \tag{B6}$$

$$[\,\kappa_a \;\equiv\; (1/\alpha - 1)[1-\alpha(1+r^F)^{-1}]\sigma[\sigma + (1-\sigma)\theta]^{-1}\,]$$

(B1)-(B5) are the direct counterparts of, respectively, (27), (23), (8), (25) and (24) in the main text. (B6) can be recognised as a form of the New Keynesian Phillips Curve (NKPC) equation, and it arises from combining the price-setting equation (12) with the price index formula (13), while also endogenising the wage by equating labour supply and demand as given by (7) and (20). The presence of $(y_t, c_t, \hat{\rho}_t)$ as separate variables in this raw form of the NKPC equation is due to the economy being open.

We now show that this equation system can be separated into two sub-systems, one of which is independent of the other. If (B4) is used to eliminate $\hat{r}_{t+1}$ from (B3), we obtain:

$$c_t - \gamma\hat{\rho}_t \;=\; c_{t+1} - \gamma\hat{\rho}_{t+1} + \zeta(v_{t+1} + d_{t+1}) \,. \tag{B7}$$

Notice that this is a first-order difference equation in the composite variable, $c_t - \gamma\hat{\rho}_t$. However, (B2) shows that $c_t - \gamma\hat{\rho}_t$ has a simple negative relationship to the trade balance, b_t. Hence we can re-write (B7) as:

$$b_t \;=\; b_{t+1} \;-\; \zeta(1-\gamma)(v_{t+1} + d_{t+1}) \,. \tag{B8}$$

This is the same as (32) in the main text. As noted there, (B5) and (B8) together form a self-contained second-order equation system in (b_t, v_t). Notably, this sub-system is independent of price stickiness: Calvo's parameter for the probability of a firm being unable to adjust price, α, is absent from it.

Next we show how the second sub-system of equations can be re-written in terms of the output gap. The output gap is the difference between y_t and the natural output level, y_t^N, so we need to establish the determinants of y_t^N. Under fully flexible prices (i.e. the hypothetical circumstances which determine y_t^N), the relationships (27), which we previously derived just for the steady state, in fact hold in every period. Once log-linearised, we may combine these with (B2) to express y_t^N as a function only of b_t:

$$y_t^N \;=\; \delta(b_t + g_t^H) \,. \qquad\qquad [\,\delta \;\equiv\; \{(\theta-1)/\theta\psi + 1/\sigma\}^{-1}] \tag{B9}$$

This is the same as (35) in the main text. It is notable that y_t^N is determined solely by the (b_t,v_t) sub-system (and by g_t^H). Given that y_t^N is the flexible-price output level and that, as remarked above, the (b_t, v_t) sub-system is independent of price stickiness whereas the other sub-system is not, we should in fact expect y_t^N to be determined solely within the former.

To re-write the NKPC equation, (B6), in terms of the output gap, we first replace $(y_t, c_t, \hat{\rho}_t)$ in (B6) by (y_t, b_t, g_t^H). This can be done by solving (B1) and (B2) simultaneously in order to express $(c_t, \hat{\rho}_t)$ as functions of (y_t, b_t, g_t^H), and then substituting the results into

(B6). We next replace y_t by $\tilde{y}_t + y_t^N$ and use (B9) to eliminate y_t^N. This causes b_t and g_t^H to drop out of the equation, so that it reduces to:

$$\pi_t^H = (1+r^F)^{-1}\pi_{t+1}^H + \kappa\tilde{y}_t, \qquad [\kappa \equiv \delta^{-1}\kappa_a] \tag{B10}$$

which is the same as (33) in the main text.

Lastly, the 'aggregate demand' function in the main text, (34), is obtained by combining (B1) and (B2) to eliminate c_t, replacing y_t by $\tilde{y}_t + y_t^N$, and then using (B9) to substitute out y_t^N.

Appendix C

We may rewrite the difference equation system comprised of (31) and (32) in matrix form as:

$$\begin{bmatrix} b_{t+1} \\ v_{t+1} \end{bmatrix} = \begin{bmatrix} 1+\zeta(1-\gamma) & \zeta(1-\gamma)(1+r^F) \\ 1 & 1+r^F \end{bmatrix} \begin{bmatrix} b_t \\ v_t \end{bmatrix} + \begin{bmatrix} \zeta(1-\gamma)d \\ 0 \end{bmatrix}. \tag{C1}$$

Let an eigenvalue of the coefficient matrix in (C1) be denoted by λ. λ is determined by the characteristic equation of the matrix, namely:

$$\lambda^2 - [1+\zeta(1-\gamma)+(1+r^F)]\lambda + (1+r^F) = 0. \tag{C2}$$

This has the structure:

$$\lambda^2 - (a+b)\lambda + b = 0, \tag{C3}$$

where $a \equiv 1+\zeta(1-\gamma) \geq 1$, $b \equiv 1+r^F > 1$. The properties of its roots, i.e. of the eigenvalues, can most easily be examined by graphing it. To do this, rearrange (C3) as:

$$\lambda^2 = (a+b)\lambda - b. \tag{C4}$$

The left-hand side (LHS) and right-hand side (RHS) expressions are both simple functions of λ. These can readily be sketched as in Figure C1.

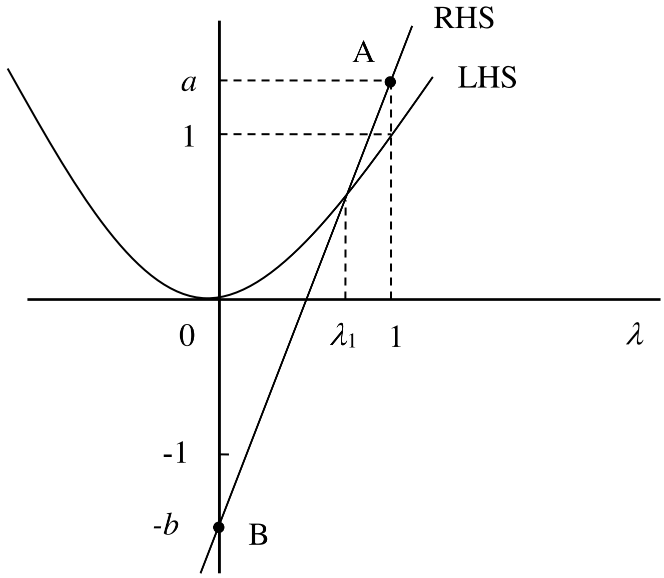

Figure C1

It is easy to see from (C4) that the LHS function is a parabola which passes through the points (0,0) and (1,1). On the other hand, the RHS function is a straight line which passes through the points (1,a) (labelled A), and (0,-b) (labelled B). Given that points A and B lie on opposite sides of the parabola, it follows that the two loci must intersect. Hence the eigenvalues are real, rather than complex, numbers. Given the location of points A and B, it is also clear that one intersection of the two loci must occur for λ in the range (0,1), while there will be another intersection (not shown) for λ in the range (1,∞). Hence, as asserted in the main text, we have one eigenvalue inside, and one outside, the unit circle.

Next consider how changes in the parameters (a,b) affect the size of the smaller eigenvalue, λ_1. Note that both a and b can vary between 1 and ∞. In particular, $q = 1$ implies $\zeta = 0$ and hence $a = 1$; while as q falls towards zero, ζ, and hence a, increase towards infinity. Meanwhile, $r^F = 0$ implies $b = 1$; while as r^F rises towards infinity, so does b. (A rise in r^F in addition increases ζ, and thus also increases a.) From Figure C1, it is easy to see that as a increases from 1 to ∞, the line 'RHS' pivots anti-clockwise about point B (assuming unchanged b), and hence λ_1 falls from 1 to 0. λ_1 is therefore an increasing function of q, tending to 0 as q tends to 0, and tending to 1 as q tends to 1. On the other hand, as b increases from 1 to ∞, the RHS line pivots anti-clockwise about point A (assuming unchanged a). Hence λ_1 rises towards 1.

In the main text, the relationship of b_t to v_t in the perfect foresight solution is given by (38). The coefficient which appears in this – η_b, say – comes from the normalised stable eigenvector of the matrix in (C1), i.e. the eigenvector associated with λ_1. η_b therefore satisfies:

$$\begin{bmatrix} 1+\zeta(1-\gamma)-\lambda_1 & \zeta(1-\gamma)(1+r^F) \\ 1 & 1+r^F-\lambda_1 \end{bmatrix} \begin{bmatrix} \eta_b \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{C5}$$

The expression for η_b in (38) is taken from the second equation in (C5).

Appendix D

In this sub-section we are not concerned with changes in government spending, so we set $g_t^H = 0$ for all t. Substituting (34) into (33) and using the definitions of $(\pi_t^H, \hat{\rho}_t)$, we obtain a second-order difference equation in p_t^H:

$$(1+r^F)^{-1} p_{t+1}^H - [(1+r^F)^{-1}+1+\kappa] p_t^H + p_{t-1}^H = \kappa[\gamma(1-\gamma)^{-1}+\delta] b_t \tag{D1}$$

The evolution of b_t in this equation is given by (38), which can be re-written as:

$$b_t = (\lambda_1 - 1 - r^F) v_t - (1-\lambda_1) d, \tag{D2}$$

while the evolution of v_t, as also given by (38), can equivalently be expressed as:

$$v_{t+1} = \lambda_1 v_t - (1-\lambda_1) d. \tag{D3}$$

We may now use (D2) in (D1) and combine this with the definition $p_t^{HP} \equiv p_{t-1}^H$ and with (D3) to obtain the following system of three simultaneous first-order difference equations:

$$\begin{bmatrix} p_{t+1}^H \\ p_{t+1}^{HP} \\ v_{t+1} \end{bmatrix} = \begin{bmatrix} 1+(1+r^F)(1+\kappa) & -(1+r^F) & (1+r^F)\kappa[\gamma(1-\gamma)^{-1}+\delta](\lambda_1-1-r^F) \\ 1 & 0 & 0 \\ 0 & 0 & \lambda_1 \end{bmatrix} \begin{bmatrix} p_t^H \\ p_t^{HP} \\ v_t \end{bmatrix}$$

$$+ \begin{bmatrix} -(1+r^F)\kappa[\gamma(1-\gamma)^{-1}+\delta](1-\lambda_1)d \\ 0 \\ -(1-\lambda_1)d \end{bmatrix} \tag{D4}$$

Again using λ as the general symbol for an eigenvalue, the characteristic equation of the coefficient matrix in (D4) is:

$$(\lambda-\lambda_1)\left\{\lambda^2 \;-\; [1+(1+r^F)\kappa+(1+r^F)]\lambda \;+\; (1+r^F)\right\} \;=\; 0 \tag{D5))}$$

$\lambda = \lambda_1$ is clearly one solution of this, confirming that the system (D4) inherits one of its eigenvalues from the (b_t, v_t) sub-system. The other two eigenvalues – λ_2 and λ_3 – are thus the solutions of:

$$\lambda^2 \;-\; [1+(1+r^F)\kappa+(1+r^F)]\lambda \;+\; (1+r^F) \;=\; 0\,. \tag{D6}$$

The properties of λ_2 and λ_3 may be determined by noting that the quadratic equation (D6) has a similar structure to (C2), already studied above. In particular, (D6) can also be written in the form (C3), where 'b' is defined as before while 'a' is now re-defined as $1+(1+r^F)\kappa$. Despite this re-definition it is still true that a and b can take any values in the interval $(1,\infty)$. It therefore follows that the characterisation performed in Appendix C can also be applied here. This means that (λ_2,λ_3) are real rather than complex, and that the smaller of them (λ_2) lies in the interval (0,1), while the larger (λ_3) lies in the interval $(1,\infty)$. Hence, as asserted in the main text, two eigenvalues of the system (D4) lie inside the unit circle while one lies outside.

Consider next how λ_2 varies with α. For α arbitrarily close to 1, κ is arbitrarily close to zero (see definition of κ), and hence a is arbitrarily close to 1. As α falls towards zero, κ increases towards infinity, and hence a increases towards infinity. The diagrammatic analysis in Appendix C then implies that λ_2 is an increasing function of α, tending to 0 as α tends to 0, and tending to 1 as α tends to 1. λ_2 as a function of α is thus quite similar to λ_1 as a function of q.

The first step in deriving (39), giving $\tilde{y}_0$ as a function of d, is to obtain p_0^H as a function of d. To do this, we appeal to the saddlepath solution of the system (D4), relating the $t = 0$ value of the non-predetermined variable p_t^H to the $t = 0$ values of the two predetermined variables (p_t^{HP}, v_t). Written in a general form, this saddlepath solution is:

$$p_0^H - p^H \;=\; \left[(b_{32}^N - b_{31}^N)(p_0^{HP} - p^{HP}) \;+\; (b_{21}^N - b_{22}^N)(v_0 - v)\right]\left[b_{21}^N b_{32}^N - b_{31}^N b_{22}^N\right]^{-1}, \tag{D7}$$

where the b_{ij}^N's are elements of the matrix of normalised eigenvectors associated with the coefficient matrix in (D4). More precisely, the *j*th column of the eigenvector matrix is the eigenvector corresponding to the *j*th eigenvalue, with the elements of the first row normalised to unity. (D7) is an application and slight adaptation of the formula in Blanchard and Kahn (1980).

The b_{ij}^N coefficients in (D7) may be evaluated from the matrix equations which define the normalised eigenvectors. The eigenvector associated with λ_1 is given by:

$$\begin{bmatrix} 1+(1+r^F)(1+\kappa)-\lambda_1 & -(1+r^F) & (1+r^F)\kappa[\gamma(1-\gamma)^{-1}+\delta](\lambda_1-1-r^F) \\ 1 & -\lambda_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ b_{21}^N \\ b_{31}^N \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

From this we may solve for (b_{21}^N, b_{31}^N) as:

$$b_{21}^N = 1/\lambda_1,$$

$$b_{31}^N = \frac{\lambda_1 + (1+r^F)/\lambda_1 - [1+(1+r^F)(1+\kappa)]}{(1+r^F)\kappa[\gamma(1-\gamma)^{-1}+\delta](\lambda_1-1-r^F)}.$$

The eigenvector associated with λ_2 is given by:

$$\begin{bmatrix} 1+(1+r^F)(1+\kappa)-\lambda_2 & -(1+r^F) & (1+r^F)\kappa[\gamma(1-\gamma)^{-1}+\delta](\lambda_1-1-r^F) \\ 1 & -\lambda_2 & 0 \\ 0 & 0 & \lambda_1-\lambda_2 \end{bmatrix} \begin{bmatrix} 1 \\ b_{22}^N \\ b_{32}^N \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

From this we may solve for (b_{22}^N, b_{32}^N) as:

$$b_{22}^N = 1/\lambda_2,$$

$$b_{32}^N = 0.$$

If we now substitute these expressions for $(b_{21}^N, b_{22}^N, b_{32}^N)$ (but not yet the expression for b_{31}^N) into (D7) we obtain:

$$p_0^H - p^H = \lambda_2(p_0^{HP} - p^{HP}) + \frac{\lambda_1-\lambda_2}{\lambda_1 b_{31}^N}(v_0 - v). \tag{D8}$$

To assist with interpreting the somewhat complex expression for b_{31}^{N}, notice that the characteristic equation (D6), when evaluated at $\lambda = \lambda_2$, can be rearranged to yield:

$$\kappa = (1+r^F)^{-1}\lambda_2^{-1}(1-\lambda_2)(1+r^F-\lambda_2). \tag{D9}$$

Using this to eliminate κ from the expression for b_{31}^{N}, we then have, after simplification:

$$b_{31}^{N} = \frac{(\lambda_1-\lambda_2)(1+r^F-\lambda_1\lambda_2)}{\lambda_1(1-\lambda_2)(1+r^F-\lambda_1)(1+r^F-\lambda_2)[\gamma(1-\gamma)^{-1}+\delta]}. \tag{D10}$$

We may now insert this into (D8). Further, given that the pre-shock steady state is the 'reference' steady state, $p_0^{HP} = v_0 = 0$. As regards the post-shock steady state values, we know that v = -d (see main text), while p^H (= p^{HP}) can be calculated as:

$$p^H = -[\gamma(1-\gamma)^{-1}+\delta]r^F d. \tag{D11}$$

Substituting these into (D8) and further simplifying, we then arrive at:

$$p_0^H = \frac{(1+r^F)(1-\lambda_1)(1-\lambda_2)^2}{1+r^F-\lambda_1\lambda_2}[\gamma(1-\gamma)^{-1}+\delta]d. \tag{D12}$$

Having solved for p_0^H, the last step is to use this in (34), together with the solution for b_0 (from (38)), to obtain an expression for $\tilde{y}_0$. After simplification this yields (39) in the main text.

Appendix E

In order to obtain the final-form solution for $\tilde{y}_t$ ((40) in the main text), we first need the final-form solutions for the state variables (p_t^H, p_t^{HP}, v_t). These can then be used in the 'aggregate demand' equation, (34) (in which we recall that $\hat{\rho}_t = -p_t^H$, while b_t is uniquely linked to v_t via (38)). In fact the final-form solution for v_t, being part of the separable sub-system in (b_t,v_t), was already given in (38).

The final-form solutions for (p_t^H, p_t^{HP}, v_t), which are the solutions of the matrix difference equation (D4), are in general given by:

$$p_t^H - p^H = w_1\lambda_1^t + w_2\lambda_2^t, \tag{E1}$$

$$p_t^{HP} - p^{HP} \; = \; w_1 b_{21}^N \lambda_1^t \; + \; w_2 b_{22}^N \lambda_2^t \, , \tag{E2}$$

$$v_t - v \; = \; w_1 b_{31}^N \lambda_1^t \; + \; w_2 b_{32}^N \lambda_2^t \, . \tag{E3}$$

Values for the b_{ij}^N coefficients were presented in Appendix D. The remaining unknowns are the weights (w_1,w_2). These are determined by the initial conditions for the two predetermined state variables, (p_t^{HP}, v_t). As explained in the main text, under our assumed policy experiment these initial conditions are $v_0 = p_0^{HU} = 0$. If we apply these to (E2) and (E3) we may now solve for (w_1,w_2) as:

$$w_1 \; = \; (1/b_{31}^N)d \, ,$$

$$w_2 \; = \; \lambda_2 \left\{ [\gamma(1-\gamma)^{-1} + \delta] r^F \; - \; 1/(\lambda_1 b_{31}^N) \right\} d \, .$$

(Here b_{31}^N has not yet been substituted out, but this can be done using (D10).)

As just noted, from (34) and (38) we may write:

$$\tilde{y}_t \; = \; - \; p_t^H \; - \; [\gamma(1-\gamma)^{-1} + \delta][(\lambda_1 - 1 - r^F) v_t \; - \; (1-\lambda_1)d] \, .$$

Using (E1) and (E3) in this, together with the expressions already provided for their constituent terms, and simplifying the resulting formula, we then arrive at (40) in the main text.

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